of coefficients a_i involved. Therefore, some form of directed search will be required. However, a consequence of system nonlinearity is that (unlike the situation for linear systems) systematic "hill-climbing" procedures can converge to a local, rather than the required global, maximum. Not only must the user satisfy himself that this problem has been overcome, but he will be required to satisfy the certification authority that this is so. Options open to the user include comparing results from searches which start from several different sets of initial conditions. A more comprehensive technique is to combine systematic hill-climbing with the incorporation of a degree of randomization, to produce a stochastic-search method (simulated annealing[9]) that has been widely used in the fields of statistical physics, image processing, and artificial intelligence.

References

[1]Jones, J. G., "On the Implementation of Power-Spectral Procedures by the Method of Equivalent Deterministic Variables—Part 1, Analytical Background," TM FS(F)485, Royal Aerospace Establishment, Farnborough, England, UK, July 1982.

[2]Jones, J. G., "An Equivalence Between Deterministic and Probabilistic Design Criteria for Linear Systems," *Journal of Sound and Vibration*, Vol. 125, Pt. 2, 1988, pp. 341–356.

[3]Jones, J. G., "Formulation of Design Envelope Criterion in Terms of Deterministic Spectral Procedure," TM SS9, Royal Aerospace Establishment, Farnborough, England, UK, Jan. 1990.

[4]Papoulis, A., "Maximum Response with Input Energy Constraints and the Matched Filter Principle," *IEEE Transactions on Circuit Theory*, Vol. CT-17, No. 2, 1970, pp. 175–182.

[5]Rice, S. O., "Mathematical Analysis of Random Noise," *Selected Papers on Noise and Stochastic Processes*, 1st ed., Dover, New York, 1954, 133–294.

[6]Anon., *Federal Airworthiness Regulations, Part 25, Airworthiness Standards: Transport Category Airplanes*, FAA, Washington, DC.

[7]Anon., *Joint Airworthiness Requirements, JAR-25, Large Aeroplanes*, Civil Aviation Authority, Cheltenham, England, UK.

[8]Vinnicombe, G., Hockenhull, M., and Dudman, A. E., "Gust Analysis of an Aircraft with Highly Nonlinear Systems Interaction," AIAA/ASME/AHS/ASC 30th Structural Dynamics and Materials Conf., AIAA Paper 89-1377, Mobile, AL, April 1989.

[9]Aarts, E., and Korst, J., *Simulated Annealing and Boltzmann Machines*, Wiley, Chichester and New York, 1989, pp. 1–200.

Coupled Deflection and Rotation of Anisotropic Open-Section Composite Stiffeners

Lawrence C. Bank* and Emmanuel Cofie†
Catholic University of America, Washington, D.C. 20064

Introduction

THIN-WALLED open-section composite material stiffeners of the *T*, *I*, and *J* geometries are routinely used in stiffened-panel construction for structural components in aerospace vehicles.[1] In certain applications, the flange and web elements of the stiffener may need to be constructed of anisotropic panels of composite materials.[2] An analytical method is outlined in this note to perform preliminary hand-calculation analysis of open-section composite stiffeners (beams) that are composed of flanges and webs having anisotropic lay-ups. The predictions of the proposed one-dimensional analytical model are compared with finite-element results using the NISA II composites finite-element code. Analytical beam-type theories for thin-walled composite beams have concentrated on modeling closed cross-sections for rotor blade applications.[3] Beam finite elements have been developed from theories of this type. The analysis of thin-walled open-section anisotropic stiffeners using finite-element analysis, has typically been performed by several authors using specialized beam elements, often based on the Vlasov beam theory.[2,4,5–8] Significant effort has been devoted to the problem of buckling and postbuckling of thin-walled laminated composite structures.[9] Recently Bank[10] and Bank and Cofie[11,12] proposed a one-dimensional theory based on a modified classical beam theory for the linear analysis of generally anisotropic open-section composite beams. The analytical theory, which permits hand-calculation of beam deformations, is intended for preliminary design studies. The theory allows the structural designer to obtain a physical feel for the behavior of anisotropic composite beams which should aid in the intuitive understanding of the behavior of composite structures.

In the theory, open-section stiffeners constructed of laminated panels of composite materials are considered. The panels can be layed-up to have in-plane anisotropy or may be in-plane orthotropic or quasi-isotropic. Each panel of the cross-section may have different mechanical properties, such that a *T* stiffener with an anisotropic flange and an orthotropic web[2] can be analyzed. The analytical theory and calculation method are described in detail elsewhere.[10–12] The fundamental assumptions of classical beam theory are modified to account for anisotropic coupling effects in the proposed analytical theory. As is commonly assumed[13] in the analysis of composite beam structures, only the in-plane stiffness of the panels is assumed to contribute to the overall beam behavior. The cases of pure bending and transverse loading are considered separately. The transverse deflection calculation is different for the two cases since the effect of shear deflection is included for transverse loading but not for pure bending. The procedure for finding the out-of-plane displacement, is however, the same for the two cases.

Modified Beam Theory

Pure Bending

Pure bending of a composite beam subjected to couples at its ends applied in the plane of symmetry of the cross-section is considered. Each panel of the beam is considered to be subjected to a linear stress distribution. Due to the anisotropically induced shear strain in the web and flange panels of the thin-walled section, the beam will deform both in-plane (transverse deflection) and out-of-plane (rotation and lateral deflection), in addition to its conventional transverse deflection related to the axial (bending) strain. If the beam is composed of panels having different mechanical properties, the appropriate mechanical constants for the panel must be used. For the calculation of the overall beam deformation a transformed section approach is used.[14,15] The total deflection y_t, of the beam is given as the sum of a bending deflection y_b, and anisotropic deflection y_a. The anisotropic deflection, due to the induced shear strain component in the web of the section, is found together with the bending deflection utilizing a modification of the Timoshenko beam theory.

Transverse Loading

Symmetric bending of a beam subjected to loading transverse to its cross-sectional plane is considered. The loads may

---

Presented as Paper 91-1022 at the AIAA 32nd Structures, Structural Dynamics, and Materials Conference, Baltimore, MD, April 8–10, 1991; received May 9, 1991; revision received Jan. 31, 1992; accepted for publication Feb. 15, 1992.

*Associate Professor, Department of Civil Engineering. Member AIAA.

†Graduate Student, Department of Civil Engineering. Student Member AIAA.

be concentrated or distributed. The effect of warping is not considered. For the statically determinate beams, no restraint against overall axial displacement of the beam exists and an additional anisotropically induced overall beam deformation component—axial displacement—is obtained. This deformation component has been shown to be small and is therefore neglected. The overall beam deflection y_t, now consists of three components: 1) the bending deflection y_b; 2) the anisotropic deflection y_a; and 3) a shear deflection y_s. The coefficient k, used in calculating the shear deflection is obtained using a method developed by Bank.[15] As stated earlier, a transformed section approach can be used for beams composed of different materials.

Rotation

Due to the anisotropically induced shear strains in the flanges of the beam, the cross-section will displace out-of-plane. This out-of-plane displacement will, in general, cause the cross section to rotate and deflect laterally. To obtain the rotation and the lateral deflection, the rate-of-twist Θ of the section is needed. It is found[12] that the rate-of-twist $\Theta(x)$ (i.e., twist per unit length) has the same functional form as $M(x)$, the bending moment. The total twist or rotation of the section ϕ is found by integration of the rate-of-twist with a properly specified boundary condition, such as $\phi(x = x_0) = \phi_0$. It is found[12] that the total twist (rotation) $\phi(x)$ has a functional form one order higher than that of the bending moment $M(x)$; (i.e., if the bending moment is constant, as in the case of pure bending, the rotation will have a linear variation along the beam length).

Lateral Deflection

The lateral deflection u_z of any point on the cross section is found by integration of the anisotropic shear strains. The lateral deflection of a point will depend on the height of the point from the center-of-twist. It should be noted that the same boundary condition must be used for both the top and bottom flanges and that this boundary condition must be compatible with the boundary condition used in calculating the twist. The lateral deflection of a point on the web of the section is found from geometry since the cross section is assumed to be rigid. Alternatively, the lateral deflection of any point can be found from the overall twist of the section.

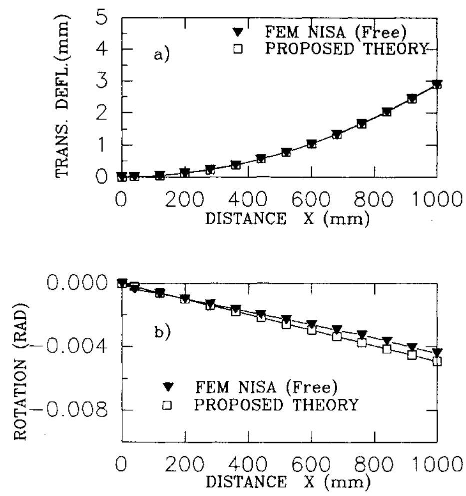

Fig. 1 a) Transverse deflection, and b) rotation vs length for example 1.

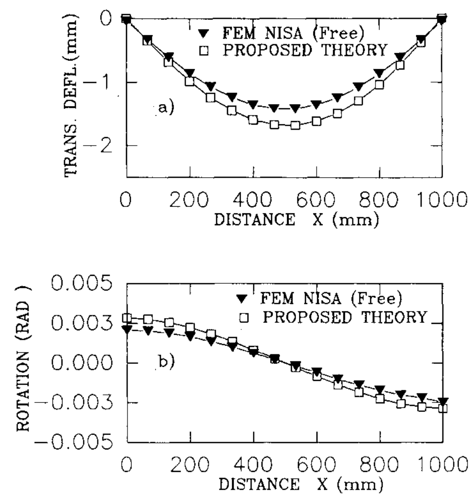

Fig. 2 a) Transverse deflection, and b) rotation vs length for example 2.

Numerical Examples

For the numerical studies the NISA II COMPOSITES finite-element code was used.[16] The examples that follow demonstrate the analytical theory and the numerical results. T300/5208 graphite/epoxy is used as a typical composite material. The properties of a unidirectional ply are $E_x = 181$ GPa, $E_y = 10.3$ GPa, $E_s = G_{xy} = 7.17$ GPa, $\nu_{yx} = 0.280$. Laminated panels 2-mm thick with on-axis (orthotropic) or off-axis (anisotropic) unidirectional plies are used to construct the thin-walled beams. More complicated anisotropic lay-ups can be analyzed. Mechanical properties (in effective engineering form) for the off-axis laminates are obtained using conventional lamination theory.[17] Three examples are presented to demonstrate the theory and the finite-element results. The examples were selected to demonstrate different aspects of the theory. Additional examples and parametric studies are given in Ref. 11. Details can be found in Ref. 12.

Example 1

A 1000-mm long thin-walled cantiliver T section loaded by a concentrated tip-moment $M_z = 10^5$ N/mm at its right end is considered. The T section has a 50-mm-wide 15-deg off-axis anisotropic top flange and 70-mm-high 0-deg orthotropic web. Fig. 1a shows the transverse deflection y along the length of the beam and Fig. 1b shows the rotation ϕ along the beam length. Results of the finite-element calculations are also shown. For the finite-slement analysis the rotation of the node at the web/flange intersection is shown for convenience. Note in this case that the transverse deflection is only due to the bending deflection component since the web is orthotropic and will have no anisotropically induced shear strain. Note also that the linear variation of the twist is confirmed by the finite-element results.

Example 2

A 1000-mm-long thin-walled simply supported T section loaded by a uniformly distributed load $q(x) = 1$ N/mm along its entire length. The beam is simply supported at its two ends $[y_t(0) = y_t(1000) = 0]$ and in addition is restrained from twisting at its midspan $[\phi(500) = 0]$. The T section has a 50-mm-wide 15-deg off-axis anisotropic top flange and a 70-mm-high 15-deg off-axis anisotropic web. Fig. 2a shows the transverse deflection y_t along the length of the beam and Fig. 2b shows the rotation ϕ along the beam length. Note the same form of the rotation curve in Fig. 2b is obtained from the theoretical and the finite element results. Also note that the transverse displacement is not symmetric with respect to the center of the span.

Example 3

A 1000-mm-long thin-walled cantilever T section loaded by a concentrated tip-load $P_y = 100$ N at its right end is considered. The T section has a 50-mm-wide 15-deg off-axis anisotropic top flange, a 70-mm-high 15-deg off-axis anisotropic web, and a 50-mm-wide 60-deg off-axis anisotropic bottom flange. Fig. 3a shows the transverse deflection y along the

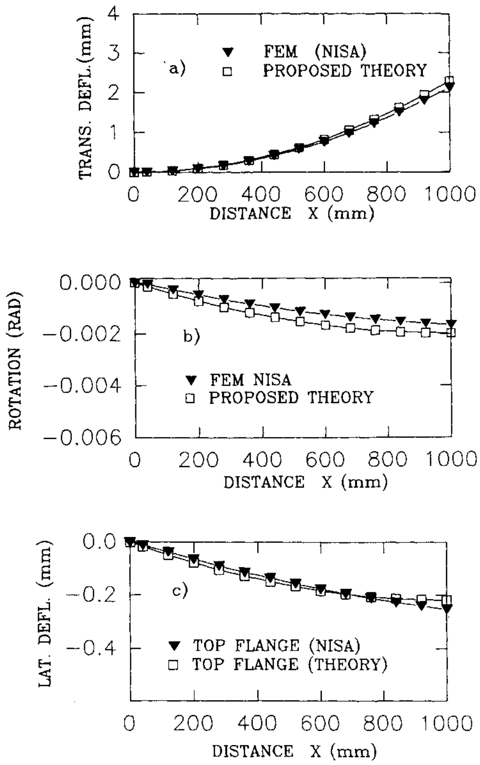

Fig. 3 a) Transverse deflection, b) rotation, and c) lateral deflection vs length for example 3.

length of the beam; Fig. 3b shows the rotation ϕ along the beam length along the length of the beam. Results of the finite-element calculations are also shown. Fig. 3c shows the lateral deflection of the top flange u_z^t along the length of the beam. The convergence of the results at the loaded end (right side) is due to a kinematic constraint condition used to enable concentrated loading at the beam end.

Conclusion

The proposed analytical theory is the first hand-calculation-type theory to be presented which can predict out-of-plane coupled deformation of transversely loaded composite beams. Numerical studies confirm the coupled deformation modes. Agreement between the theory and the finite-element results is remarkably good considering the simplicity of the theory. Parametric studies have shown maximum differences between the theory and the finite-element results to be up to approximately 15% for transverse deflections and up to approximately 25% for rotations and lateral deflections. The theory should be useful to designers wishing to perform preliminary studies of open-section stiffener sections. The theory is currently being extended to predict behavior under torsional loading, the next step in the development of a generally anisotropic beam element.

References

[1]Starnes, J. H., Knight, N. F., Jr., and Rouse, M., "Postbuckling Behavior of Selected Flat Graphite Epoxy Panels Loaded in Compression," *AIAA Journal*, Vol. 23, No. 3, 1985, pp. 1236–1246.

[2]Stein, M., "Postbuckling of Eccentric Open-Section Stiffened Composite Panels," *Proceedings of the 29th Structures, Structural Dynamics and Materials Conference*, AIAA, Washington, DC, April 18–20, 1988, pp. 57–61.

[3]Hodges, D. H., "Review of Composite Rotor Blade Modeling," *AIAA Journal*, Vol. 28, No. 3, 1990, pp. 561–565.

[4]Nagarajan, S., and Zak, A. R., "Finite Element Model for Orthotropic Beams," *Computers & Structures*, Vol. 20, Nos. 1–3, 1985, pp. 443–449.

[5]Bauld, J. R., and Teng, L. S., "A Vlasov Theory for Fiber-Reinforced Beams with Thin Walled Open Cross Sections," *International Journal of Solids and Structures*, Vol. 20, No. 3, 1984, pp. 277–297.

[6]Gupta, R. K., Venkatesh, A., and Rao, K. P., "Finite Element Analysis of Laminated Anisotropic Thin Walled Open Section Beam," *Composite Structures*, Vol. 3, No. 1, 1985, pp. 19–31.

[7]Lo, P. K., and Johnson, E. R., "One-Dimensional Analysis of Filamentary Composite Beam Columns with Thin-Walled Open Sections," *Composites '86: Recent Advances in the US and Japan, Proceedings of the Japan—U.S. CCM III*, Japan Society for Composite Materials, Tokyo, Japan, 1986, pp. 405–414.

[8]Chandra, R., Stemple, A. D., and Chopra, I., "Thin Wall Composite Beam Under Bending, Torsional and Extensional Loads," *Journal of Aircraft*, Vol. 27, No. 7, 1990, pp. 619–627.

[9]Noor, A. K., and Peters, J. M., "Buckling and Postbuckling Analyses of Laminated Anisotropic Structures," *International Journal for Numerical Methods in Engineering*, Vol. 27, No. 2, 1989, pp. 383–401.

[10]Bank, L. C., "Modifications to Beam Theory for Bending and Twisting of Open-Section Composite Beams," *Composite Structures*, Vol. 15, No. 15, 1990, pp. 93–114.

[11]Bank, L. C., and Cofie, E., "A Modified Beam Theory for Bending and Twisting of Open-Section Composite Beams:—Numerical Verification," *Composite Structures*, Vol. 21, No. 1, 1992, pp. 29–39.

[12]Bank, L. C., and Cofie, E., "Coupled Deflection and Rotation of Anisotropic Open-Section Composite Stiffeners," *Proceedings of the 32nd Structures, Structural Dynamics and Materials Conference*, AIAA, Washington, DC, April 8–10, Baltimore, MD, 1991, pp. 1027–1036.

[13]Vinson, J. R., and Sierakowski, R. L., *The Behavior of Structures Composed of Composite Materials*, Martinus Nijhoff, Dordrecht, 1987, pp. 29–146.

[14]Bank, L. C., and Bednarczyk, P. J., "A Beam Theory for Thin-Walled Composite Beams," *Composites Science and Technology*, Vol. 32, No. 4, 1988, pp. 265–277.

[15]Bank, L. C., "Shear Coefficients for Thin-Walled Composite Beams," *Composite Structures*, Vol. 8, No. 1, 1987, pp. 47–61.

[16]NISA Users Manual, NISA II COMPOSITE Programs, Version 90.0, Engineering Mechanics Research Corp., Troy, MI, Chaps. 4 and 5.

[17]Tsai, S. W., and Hahn, H. T., *Introduction to Composite Materials*, Technomic Publishing, Lancaster, PA, 1980, pp. 31–113.

Viscous Subsonic Flow Computation for Wings with Flaps for High-Lift

H. N. V. Dutt*
National Aeronautical Laboratory, Bangalore, India
and
K. Jacob†
DLR Goettingen, Germany

Introduction

ANALYSIS of viscous flow over high-lift systems for large aspect ratio transport wings is one of the important problems of aircraft aerodynamics. The problem of viscous flow over clean wings was considered in Ref. 1. In the present Note, the method of Ref. 1 has been extended for analysis of multielement wings comprised of multicomponent airfoils at high-lift and includes a model for ground effect, compressibility, trailing-edge separation, and curved basic flow.

First attempts to validate the method by comparing computed results with measurements are reported. Within the

Received Aug. 12, 1991; revision received Nov. 22, 1991; accepted for publication Dec. 16, 1991.

*Scientist, Experimental Aerodynamics Division.
†Scientist, Theoretical Fluid Mechanics.